Exam Seat No:_______________ Enrollment No:___________________

C.U.SHAH UNIVERSITY

Wadhwan City

Subject Code 4TE01EMT1

Summer Examination-2014 **Date: 27/05/2014**

Subject Name: Engineering Mathematics-I

Branch/Semester:- B.Tech/I **Time:10:30 To 1:30**

Examination: Remedial

Instructions:-

(1) Attempt all Questions of both sections in same answer book / Supplementary
(2) Use of Programmable calculator & any other electronic instrument is prohibited.
(3) Instructions written on main answer Book are strictly to be obeyed.
(4) Draw neat diagrams & figures (If necessary) at right places
(5) Assume suitable & Perfect data if needed

SECTION-I

Q-1 a) Suppose that the inequalities $1-\frac{x^2}{2} < \frac{x\sin x}{2-2\cos x} < 1$ hold for all values of x close to zero. What if anything, does this tell you about $\lim_{x\to 0}\frac{x\sin x}{2-2\cos x}$? (01)

b) Express $\frac{3+7i}{2+5i}$ in standard form $x+iy$. (01)

c) Find imaginary part of $z=\frac{5}{(1-i)(1+i)}$. (01)

d) Find modulus and principle argument of $1-i\sqrt{3}$. (01)

e) For what values of a is $f(x)=\begin{cases} x^2-1, & x<3 \\ 2ax, & x\geq 3\end{cases}$ continuous at every x? (02)

f) Evaluate $\lim_{x\to 0}\frac{\sin 2x}{x}$. (01)

Q-2 a) State and prove De' Moivre's theorem. (05)

b) Prove that $(1+i)^n+(1-i)^n = 2^{\frac{n}{2}+1}\cos\left(\frac{n\pi}{4}\right)$. (05)

c) Find the fourth roots of unity and sketch them on the unit circle. (04)

OR

Q-2 a) State and prove triangular inequality for complex numbers. (05)

b) Prove that (05)

$$(1+\cos\theta+i\sin\theta)^n+(1+\cos\theta-i\sin\theta)^n = 2^{n+1}\cos^n\left(\frac{\theta}{2}\right)\cos\left(\frac{n\theta}{2}\right)$$

c) Solve $x^4-x^3+x^2-x+1=0$. (04)

Q-3 a) Check whether the series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\cdots$ is (i) absolutely convergent or (ii) conditionally convergent (05)

b) Test the convergence of $\sum_{n=1}^{\infty}\frac{n}{(n^3+1)^{\frac{5}{7}}}$. (05)

c) Evaluate $\lim_{x \to 0}\left(\frac{1}{\sin x}-\frac{1}{x}\right)$. (04)

OR

Q-3 a) Find radius of convergence and interval of convergence of the power series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^{2n-1}}{2n-1}$. (05)

b) Test the convergence of $\sum_{n=1}^{\infty}\left(\frac{n}{2n+5}\right)^n$. (05)

c) Evaluate $\lim_{x \to 0} \log_{\tan x}\tan 2x$. (04)

SECTION-II

Q-4 a) Find equation of tangent plane and normal line of the surface $x^2+y^2+z-9=0$ at the point $(1,\ 2,\ 4)$. (02)

b) Show that $f(x,y)=-\frac{x}{\sqrt{x^2+y^2}}$ has no limit as $(x,\ y)\to(0,\ 0)$. (02)

c) Find $\frac{\partial(x,\ y)}{\partial(r,\ \theta)}$, if $x=r\cos\theta$, $y=r\sin\theta$. (02)

d) State Maclaurin's series. (01)

Q-5 a) If $u=(x^2+y^2+z^2)^{-\frac{1}{2}}$, prove that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0$. (05)

b) If $u=f(x^2+2yz, y^2+2zx)$, prove that $(y^2-zx)\frac{\partial u}{\partial x}+(x^2-yz)\frac{\partial u}{\partial y}+(z^2-xy)\frac{\partial u}{\partial z}=0$. (05)

c) Trace the curve $y^2(2a-x)=x^3$. (04)

OR

Q-5 a) If $u=e^{ax}\tan by \log z$, verify that $\frac{\partial^3 u}{\partial x\partial y\partial z}=\frac{\partial^3 u}{\partial z\partial y\partial x}$. (05)

b) Show that $f(x,\ y)=\begin{cases}\frac{2xy}{x^2+y^2}, & (x,\ y)\neq(0,\ 0)\\ 0, & (x,\ y)=(0,\ 0)\end{cases}$ is continuous everywhere except at origin. (05)

c) Trace the curve $r^2=a^2\cos 2\theta$. (04)

Q-6 a) State Euler's theorem and if $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ then prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u$. (05)

b) Find the maxima and minima of $f(x,y)=x^3+y^3-3x-12y+20$. (05)

c) Expand $\log(1+\sin x)$ in powers of x. (04)

OR

Q-6 a) If $u=\operatorname{cosec}^{-1}\left(\frac{\sqrt{x}+\sqrt{y}}{\sqrt[3]{x}+\sqrt[3]{y}}\right)^{\frac{1}{2}}$, prove (05) that $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = \frac{1}{144} tanu\ (13 + tan^2u)$.

b) Find greatest and smallest values that the function $f(x,\ y) = xy$ takes on the ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$. (05)

c) Using Taylor's series, find the value of $\sin 44^{\circ}$. (04)

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